Inner Models for Set Theory Defined by Generalized Logics

Menachem Magidor//Hebrew University of Jerusalem

June 20, 2013

(Joint work with J. Kennedy and J. Vaananen)

The theory of inner models of Set Theory is an attempt to define inner models that will be canonical and at the same time rich enough to include interesting definable objects.

Gödels constructible universe **L** is a very canonical inner model but it misses some very canonical definable objects (e.g. $0^{\#}$). The collection of sets in **L** is defined by a series of steps L_α where the successor stage $L_{\alpha+1}$ is the collection of subsets of L_α *first order* definable in $\langle L_\alpha, \epsilon \rangle$.

What happens when one replaces *first order* in the definition above by stronger logic? It is well known (due to Myhill and Scott) that if one uses full secind order logic then one gets **HOD**, the model of the hereditarily ordinal definable sets. This is a very rich inner model but it is highly non canonical. In the talk we shall survey the models one gets by using logic which in between first order and second order. Typically they will be first order logic augmented by additional quantifiers (e.g. The Kiesler quantifier $Q_1 x$ meaning there are uncountable many x‘s such that $\ldots$, the Magidor- Malitz quantifier $Q_2 xy$ meaning "there is an uncountable set A such that for every $x, y \in A \ldots$ and the quantifier $Q^{cof}_\omega x, y$ meaning "We have a linear order with countable cofinality".